

The interplay between Weak and Strong Phases and Direct CP Violation from the Charmless B-meson Decays

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Abstract

We present a general analysis on charmless B-meson decays $B \rightarrow \pi\pi$ and πK . It is noticed that the final state interactions and inelastic rescattering effects must be significant in order to understand the consistency of the current data. By using general isospin decompositions, the isospin amplitudes and the corresponding strong phases could be extracted from a global χ^2 fit of the experimental data. We emphasize that in general there are two, rather than one, relative strong phases in the decomposition. In the assumption of two equal strong phases as considered in the literature, the current data, especially the ones concerning $B \rightarrow \pi^0 K^{0(\pm)}$ decays, will imply a large isospin amplitude $|a_{3/2}^c| > 40$, which is larger by a factor of five than the one from the naive factorization estimation. When two different strong phases are considered, all the isospin amplitudes can become, within the 1σ -level, comparable with the theoretical values. We also show that the difference between the two strong phases cannot be too large and will be restricted by the most recent upper bound of $B \rightarrow \pi^0\pi^0$ decay. In any case, the strong phases are found to be large and the branching ratio of $B \rightarrow \pi^0\pi^0$ is likely to be enhanced by an order of magnitude in comparison with the one obtained from the naive factorization approach. Direct CP violations in all decay modes are also calculated and found to be close to the sensitivity of the present experiments.

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I. INTRODUCTION

Recently the CLEO collaboration has reported measurements on the branching ratios of rare hadronic B decays $B \rightarrow \pi\pi, \pi K$ [1,2]. The data have attracted great interest from both theorists and experimentalists. The study of these channels will provide us important insights on understanding the effects of electroweak penguins (EWP) in B-system [3], and the final state interactions(FSI) [4,5], as well as extracting the weak CKM phase $\gamma = \arg(V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ [6–8,10]. It may also open a window for probing new physics [11].

From the current data, it is noticed that the branching ratio for $B \rightarrow \pi^+\pi^-$ is relatively small, $Br(B \rightarrow \pi^+\pi^-) \sim 4$ (in units of 10^{-6}). The decays for $B \rightarrow \pi^+K^-$, $\pi^- \bar{K}^0$ have almost equal decay rates, i.e., $Br(B \rightarrow \pi^+K^-) \simeq Br(B \rightarrow \pi^- \bar{K}^0) \sim 17$. While an unexpectedly large branching ratio for $B \rightarrow \pi^0 \bar{K}^0$ decay was also observed, $Br(B \rightarrow \pi^0 \bar{K}^0) \sim 14$. These measurements seem in conflict with the calculations based on the naive factorization hypotheses. For the first three decays, it was pointed out that the factorization approach may still be valid if one takes the weak phase γ to be greater than 90° [8]. With such a large γ , i.e., $\cos \gamma < 0$, the interference between tree and penguin diagrams has opposite sign in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays. Thus the negative $\cos \gamma$ will suppress the decay rate for $B \rightarrow \pi\pi$ and enhance that for $B \rightarrow \pi^+K^-$. As a consequence, the almost equal decay rates for $B \rightarrow \pi^+K^-$ and $B \rightarrow \pi^- \bar{K}^0$ decay modes indicate the dominance of strong penguin. However, the large rate for $B \rightarrow \pi^0 \bar{K}^0$ is not easily explained. Most recent analysis showed that a large final state interaction (FSI) phase would be helpful to enhance the branching ratio for the $B \rightarrow \pi^0 \bar{K}^0$ decay [4], but only considering the elastic-rescatterings remains insufficient to obtain the large central value of the data.

To understand the measured data, besides some model-dependent calculations, a model-independent approach using a single relative strong phase has also been proposed for the study of $B \rightarrow \pi\pi, \pi K$ decays [4,9,8]. Since the ordinary factorization approach suffers from the uncertainties due to hadronic matrix elements, such as the meson decay constants, B-meson form factors and so-called effective color number N_c . The model-independent analyses may be more useful as more data become available. The approach based on the isospin $SU(2)$ and approximate flavor $SU(3)$ [12] symmetries of the strong interactions has been proposed to constrain [13,14] and extract the weak phase γ [7]. It has been noticed that the ratios between the CP-averaged decay rates, such as $R = Br(B \rightarrow \pi^\pm K^0)/Br(B \rightarrow \pi^0 K^\pm)$ may provide us important information on the weak phase γ . Most recently, it has been shown that the weak phase γ may be determined through three ratios among CP-averaged decay rates of $B \rightarrow \pi^+\pi^-$, $\pi^+\pi^0$, π^-K^+ and π^-K^0 decays [10]. Where two solutions were obtained at the 1σ level, one with positive $\cos \delta$ and negative $\cos \gamma$, i.e., relative small strong phase δ and large weak phase γ , and another with negative $\cos \delta$ and positive $\cos \gamma$. The latter with positive $\cos \gamma$ seems to be favored by solutions obtained from other constraints in the standard model but appears not to be as favorable as the one with negative $\cos \gamma$ studies of all the existing charmless decays are taken in account. However, there is still no complete analysis in the literature.

In this paper, we shall give a general analysis for all 7 decay modes of $B \rightarrow \pi\pi$ and πK . For that purpose, we will start from a general model independent parameterization for all the decay amplitudes by considering both the isospin and simple diagrammatic decompositions. We will show that there are in general 15 independent variables. By assuming the $SU(3)$

relations which appropriately account for SU(3) symmetry breaking effects, it allows us to reduce the 15 independent variables into 9. They consist of 6 isospin amplitudes and 2 relative strong phase as well as one weak phase γ . Note that once the relative strong phase is zero, $B \rightarrow \pi^- \bar{K}^0$ only receives contributions from penguin-type diagram and the amplitudes of the isospin $I = 1/2$ and $I = 3/2$ amplitudes from tree-type diagrams cancel each other, which provides an additional constraint [8]. In fact, one of the isospin amplitudes becomes almost irrelevant due to the suppression factor of the CKM mixing element. With such a consideration, there are only 8 relevant unknown quantities with 5 isospin amplitudes and 3 phases. We show that the current 6 measured decay rates allow us to extract 6 unknown quantities as functions of two variable. The upper bound of the decay rate $B \rightarrow \pi^0 \pi^0$ also provides a bound for the difference between the two strong phases. Once taking the numerical value of the weak phase γ to be the one obtained from other constraints in the standard model and fixing one of the strong phases, all the other parameters can be determined. With these determined parameters, we are then able to predict the branching ratio of the $B \rightarrow \pi^0 \pi^0$ decay mode which is yet unmeasured due to the difficulty of its identification by the current detector. In addition we also present predictions for direct CP violations in all 7 decay channels of $B \rightarrow \pi\pi$, πK . In our numerical fitting, we have adopted the χ^2 analysis for the CLEO data in order to have a systematic treatment on the experimental errors.

In general, according to the Watson theorem, there are two independent relative strong phases associating with the isospin amplitudes. They are often assumed to be equal in the literatures [4,8,9]. In this work, we shall make a more general analysis with two relative strong phases. It is shown that the equal phase assumption will result in large enhancement of isospin amplitude $a_{3/2}^c$ which will be 5 times larger than the one calculated from the factorization approach. The value of the strong phase is found to be $\delta \simeq \pm 95^\circ$. These large values may imply large inelastic FSI or indicate the possible new physics effects. However, if the two strong phases are different, the value of $a_{3/2}^c$ can be lower and is comparable with the usual factorization calculations.

It is remarkable to observe that within 1σ all the 6 decay rates can be consistently fitted for a large range of the weak phase $0^\circ < \gamma < 180^\circ$ for the above two cases. It is also of interest to note that one of isospin amplitudes and the strong phases have a weak dependence on the weak phase γ . Three isospin amplitudes show a moderate dependence on the weak phase γ . Only one isospin amplitude is sensitive to γ . In particular, the fitting values for the 4 usual isospin amplitudes considered in most of the literature could still be comparable with the ones obtained by using naive factorization approach. The resulting large strong phases may be regarded as a strong indication of large FSI in $B \rightarrow \pi\pi, \pi K$ decays.

II. THE GENERAL FRAMEWORK

We begin with writing the decay amplitude of $B \rightarrow \pi\pi, \pi K$ in the following general form:

$$A^{\pi\pi(\pi K)} = \lambda_u^{d(s)} A_u^{\pi\pi(\pi K)} + \lambda_c^{d(s)} A_c^{\pi\pi(\pi K)}, \quad (1)$$

where $\lambda_u^{d(s)} = V_{ub} V_{ud(s)}^*$ and $\lambda_c^{d(s)} = V_{cb} V_{cd(s)}^*$ are the products of CKM matrix elements. The term proportional to $\lambda_t^{d(s)} = V_{tb} V_{td(s)}^*$ has been absorbed into the above two terms by using the unitarity relation, $V_{ub} V_{ud(s)}^* + V_{cb} V_{cd(s)}^* + V_{tb} V_{td(s)}^* = 0$.

we also find it useful to adopt the isospin decomposition for the decay amplitudes

$$A_{\pi^-\pi^+}^{u,c} = \sqrt{\frac{2}{3}}a_0^{u,c}e^{i\delta_0} + \sqrt{\frac{1}{3}}a_2^{u,c}e^{i\delta_2} \quad (2)$$

$$A_{\pi^0\pi^0}^{u,c} = \sqrt{\frac{1}{3}}a_0^{u,c}e^{i\delta_0} - \sqrt{\frac{2}{3}}a_2^{u,c}e^{i\delta_2} \quad (3)$$

$$A_{\pi^-\pi^0}^{u,c} = -\sqrt{\frac{3}{2}}a_2^{u,c}e^{i\delta_2} \quad (4)$$

$$A_{\pi^+K^-}^{u,c} = \sqrt{\frac{2}{3}}a_{1/2}^{u,c}e^{i\delta_{1/2}} + \sqrt{\frac{1}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}} \quad (5)$$

$$A_{\pi^0\bar{K}^0}^{u,c} = \sqrt{\frac{1}{3}}a_{1/2}^{u,c}e^{i\delta_{1/2}} - \sqrt{\frac{2}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}} \quad (6)$$

$$A_{\pi^0K^-}^{u,c} = -\sqrt{\frac{3}{2}}a_{3/2}^{u,c}e^{i\delta_{3/2}} - \frac{1}{\sqrt{2}}A_{\pi^-\bar{K}^0}^{u,c} \quad (7)$$

with

$$A_{\pi^-\bar{K}^0}^{u,c} = \sqrt{\frac{2}{3}}b_{1/2}^{u,c}e^{\delta'_{1/2}} - \sqrt{\frac{1}{3}}a_{3/2}^{u,c}e^{i\delta_{3/2}} \quad (8)$$

Where $a_I^{u,c}$ and $b_I^{u,c}$ are the isospin amplitudes and δ_I and $\delta'_{1/2}$ are the strong phases due to final state interactions. In some literatures the strong phase of isospin amplitude $b_{1/2}^{u,c}$ is assumed to be equal to the one of $a^{u,c}$ for simplicity [4,8,9]. However, in the most general case, these strong phases are not necessarily the same, since they arise from the effective Hamiltonian with different isospin. The subscript $I = 0, 2, 1/2, 3/2$ denote the isospins of the amplitudes. The advantage of the isospin decomposition allows one to use $SU(3)$ relations including leading order $SU(3)$ breaking effects. In other words, the isospin amplitudes are assumed to satisfy the following relations:

$$\begin{aligned} a_0^{u,c} &\simeq (f_\pi/f_K)a_{1/2}^{u,c}, & a_2^{u,c} &\simeq (f_\pi/f_K)a_{3/2}^{u,c} \\ \delta_0 &\simeq \delta_{1/2}, & \delta_2 &\simeq \delta_{3/2} \end{aligned} \quad (9)$$

where f_π and f_K are the π, K meson decay constants with $f_\pi/f_K \simeq 0.8$. For convenience, we define two phase differences as follows:

$$\begin{aligned} \delta &= \delta_{3/2} - \delta_{1/2} \\ \delta' &= \delta'_{1/2} - \delta_{1/2} \end{aligned} \quad (10)$$

Practically, the decay amplitudes are evaluated by calculating various Feynman diagrams. In order to see how those isospin amplitudes receive contributions from diagrams, we also present a simple diagrammatic decomposition. The diagrams can in general be classified into 6 types denoted by T (tree), C (color suppressed tree) P (QCD penguin), P_{EW} (electroweak penguin) and P_{EW}^C (color suppressed electroweak penguin) [15]:

$$A_{\pi^-\pi^+} = T + P + \frac{2}{3}P_{EW}^C \quad (11)$$

$$A_{\pi^0\pi^0} = \frac{1}{\sqrt{2}}(-C + P - P_{EW} - \frac{1}{3}P_{EW}^C) \quad (12)$$

$$A_{\pi^-\pi^0} = \frac{1}{\sqrt{2}}(-T - C - P_{EW} - P_{EW}^C) \quad (13)$$

$$A_{\pi^+K^-} = T' + P' + \frac{2}{3}P_{EW}'^C \quad (14)$$

$$A_{\pi^0\bar{K}^0} = \frac{1}{\sqrt{2}}(-C' + P' - P_{EW}' - \frac{1}{3}P_{EW}'^C) \quad (15)$$

where the primed and unprimed quantities are the amplitudes in $B \rightarrow \pi K$ and $B \rightarrow \pi\pi$ decays. They roughly differ by a factor $f_\pi/f_K \simeq 0.8$ when the $SU(3)$ flavor symmetry breaking effects are considered.

Combining the two decompositions, it is straight forward to get the following relations:

$$a_{1/2}^{u,c} e^{i\delta_{1/2}} = \frac{1}{\sqrt{6}} \left(2T' - C' + 3P' - P_{EW}' + P_{EW}'^C \right)^{u,c}, \quad (16)$$

$$a_{3/2}^{u,c} e^{i\delta_{3/2}} = \frac{1}{\sqrt{3}} \left(T' + C' + P_{EW}' + P_{EW}'^C \right)^{u,c}. \quad (17)$$

From the above equations, one may easily see the relative magnitudes among those $SU(2)$ invariant amplitudes. If the inelastic rescattering effects are small, T', C' will only contribute to the term proportional to λ_u^s . Therefore one may expect that $T'(C')^u \gg T'(C')^c$. This will lead to $a_{1/2(3/2)}^u \gg a_{1/2(3/2)}^c$. Since $a_{1/2}^c$ receives contributions from QCD penguins, while $a_{3/2}^c$ only gets contributions from EWPs, one may conclude that $a_{1/2}^c \gg a_{3/2}^c$.

To obtain relations for the isospin amplitude $b_{1/2}^{u,c}$, one needs to be careful in adopting the diagrammatic decomposition implied by the naive factorization ansatz. This is because the resulting relative strong phase is zero in the factorization approach, i.e., $\delta = \delta_{3/2} - \delta_{1/2} = 0$ and $\delta' = 0$. As a consequence,

$$A_{\pi^0K^-} = \frac{1}{\sqrt{2}}(-T' - C' - P' - P_{EW}' - \frac{2}{3}P_{EW}'^C) \quad (18)$$

$$A_{\pi^-\bar{K}^0} = P' - \frac{1}{3}P_{EW}'^C \quad (19)$$

The amplitudes with isospin $I = 1/2$ and $I = 3/2$ from tree-type graphs cancel each other in Eq.(19). Thus the total amplitude only receives contributions from penguin diagrams in this case, namely

$$A_{\pi^-\bar{K}^0}^{u,c} = \sqrt{\frac{2}{3}}b_{1/2}^{u,c} - \sqrt{\frac{1}{3}}a_{3/2}^{u,c} = \left(P' - \frac{1}{3}P_{EW}'^C \right)^{u,c} \quad (20)$$

If assuming the t -quark dominance in the penguin loops, one finds from eq.(20) that :

$$A_{\pi^-\bar{K}^0}^u \simeq A_{\pi^-\bar{K}^0}^c \quad \text{or} \quad b_{1/2}^u \simeq b_{1/2}^c + \frac{1}{\sqrt{2}}a_{3/2}^u - \frac{1}{\sqrt{2}}a_{3/2}^c. \quad (21)$$

which may be assumed for simplicity to be approximately valid after considering final state interactions with nonzero strong phases. In the numerical calculations, we have checked that the amplitude $b_{1/2}^u$ is less important due to the strong suppression of the CKM factor (for instance, even if taking $b_{1/2}^u \simeq b_{1/2}^c$, the results remain almost unchanged).

With the above analyses, let us provide an intuitive discussion on how to yield a large branching ratio for $B \rightarrow \pi^0 \bar{K}^0$ decay by appropriately choosing the isospin amplitudes. Note the fact that $\lambda_u^s \ll \lambda_c^s$, one may roughly estimate the ratio between $Br(B \rightarrow \pi^0 \bar{K}^0)$ and $Br(B \rightarrow \pi^+ K^-)$ by neglecting the terms containing the CKM factor λ_u^s

$$R = \frac{Br(B \rightarrow \pi^0 \bar{K}^0)}{Br(B \rightarrow \pi^+ K^-)} \simeq \left| \frac{\sqrt{\frac{1}{3}} a_{1/2}^c - \sqrt{\frac{2}{3}} a_{3/2}^c e^{i\delta}}{\sqrt{\frac{2}{3}} a_{1/2}^c + \sqrt{\frac{1}{3}} a_{3/2}^c e^{i\delta}} \right|^2, \quad (22)$$

with $\delta = \delta_{3/2} - \delta_{1/2}$. Once neglecting $a_{3/2}^c$, the ratio may be simply given by $R \simeq \frac{1}{2}$, which is much smaller than the central value of the data $R = 0.84$. It indicates that to enhance the decay rate of $B \rightarrow \pi^0 \bar{K}^0$, the isospin amplitude $a_{3/2}^c$ should not be neglected. Its small value may provide a sizable contribution for a large value of $\delta > \pi/2$. This is because in this case there exists a constructive interference between $a_{1/2}^c$ and $a_{3/2}^c$ in $B \rightarrow \pi^0 \bar{K}^0$ and a destructive interference in $B \rightarrow \pi^+ K^-$. The situation is quite similar to the case for a large $\gamma > \pi/2$, which is considered to enhance $B \rightarrow \pi K$ and decrease $B \rightarrow \pi\pi$ decay rates. From eq.(22), it is easily seen that the value of $a_{3/2}^c$ satisfies

$$a_{3/2}^c \geq \frac{\sqrt{2R} - 1}{\sqrt{2} + \sqrt{R}} a_{1/2}^c \simeq 0.12 a_{1/2}^c. \quad (23)$$

With the above considerations, there are only 8 relevant quantities: $a_{1/2}^{u,c}$, $a_{3/2}^{u,c}$, $b_{1/2}^c$, δ , δ' , γ , which should be constrained by six measured decay rates and one upper bound. When taking the weak phase γ as a free parameter, the rest 6 variables can be determined from 6 equations of eqs.(2-7). As the errors in the current data remain considerable large, one may not take the central values of the data to be too serious. Thus by only using the central values of the data to determine the 6 variables may not be good enough. To take into account the experimental errors in a systematic way, we shall adopt a global χ^2 (least square) analysis for the present data. This treatment allows us to obtain not only the central value but also the errors for the fitting amplitudes. Our fitting will be carried out by using the standard χ^2 analysis program package MINUIT [16].

III. RESULTS AND DISCUSSIONS

In order to compare with the values estimated from the factorization, it is necessary to explicitly see how large of the isospin amplitudes $a_{1/2,3/2}^{u,c}$ and $b_{1/2}$ from the factorization calculations, we present the relevant formulae for the $B \rightarrow \pi\pi, \pi K$ decay amplitudes with the assumption of factorization [18]:

$$A_{\pi^+ K^-} = i \frac{G_F}{\sqrt{2}} f_K F_0^{B\pi}(m_K^2) (m_B^2 - m_\pi^2) (\lambda_u^s a_1 - \lambda_t^s (a_4 + a_{10} + (a_6 + a_8) R_4)) \quad (24)$$

$$A_{\pi^0 \bar{K}^0} = -i \frac{G_F}{2} f_K F_0^{B\pi}(m_K^2) (m_B^2 - m_\pi^2) \lambda_t^s \left(a_4 + a_6 R_5 - \frac{1}{2} (a_{10} + a_8 R_5) \right)$$

$$-i\frac{G_F}{2}f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2) \left(\lambda_u^s a_2 - \lambda_t^s \frac{3}{2}(a_9 - a_7) \right) \quad (25)$$

$$A_{\pi^0 K^-} = -i\frac{G_F}{2}f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2) (\lambda_u^s a_1 - \lambda_t^s(a_4 + a_6 R_4 + (a_{10} + a_8 R_4))) \\ -i\frac{G_F}{2}f_\pi F_0^{BK}(m_\pi^2)(m_B^2 - m_K^2) \left(\lambda_u^s a_2 - \lambda_t^s \frac{3}{2}(a_9 - a_7) \right) \quad (26)$$

$$A_{\pi^- K^0}^c = -i\frac{G_F}{\sqrt{2}}f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)\lambda_t^s \left(a_4 + a_6 R_5 - \frac{1}{2}(a_{10} + a_8 R_5) \right) \quad (27)$$

where $f_{\pi,K}$ and $F^{B\pi,BK}$ are the decay constants and B -meson form factors respectively, $R_4 = 2m_K^2/(m_b - m_u)(m_u + m_s)$ and $R_5 = 2m_K^2/(m_b - m_d)(m_d + m_s)$. In the flavor $SU(2)$ limit, one has $R_4 \simeq R_5 \simeq 2m_K^2/(m_b m_s)$.

The expressions of the isospin amplitudes can be rewritten as follows:

$$a_{1/2}^u e^{i\delta_{1/2}} = \sqrt{\frac{2}{3}}A_{\pi^+ K^-}^u + \sqrt{\frac{1}{3}}A_{\pi^0 \bar{K}^0}^u \\ = r \left(\sqrt{\frac{2}{3}}(a_1 + a_4 + a_{10} + (a_6 + a_8)R_4) + \sqrt{\frac{1}{6}}(a_4 + a_6 R_5 - \frac{1}{2}(a_{10} + a_8 R_5)) \right. \\ \left. - \sqrt{\frac{1}{6}}(a_2 + \frac{3}{2}(a_9 - a_7)X) \right) \quad (28)$$

$$a_{1/2}^c e^{i\delta_{1/2}} = \sqrt{\frac{2}{3}}A_{\pi^+ K^-}^c + \sqrt{\frac{1}{3}}A_{\pi^0 \bar{K}^0}^c \\ = r \left(\sqrt{\frac{2}{3}}(a_4 + a_{10} + (a_6 + a_8)R_4) + \sqrt{\frac{1}{6}}(a_4 + a_6 R_5 - \frac{1}{2}(a_{10} + a_8 R_5)) \right. \\ \left. - \sqrt{\frac{1}{2}\frac{\sqrt{3}}{2}}(a_9 - a_7)X \right) \quad (29)$$

$$a_{3/2}^u e^{i\delta_{3/2}} = \sqrt{\frac{1}{3}}A_{\pi^+ K^-}^u - \sqrt{\frac{2}{3}}A_{\pi^0 \bar{K}^0}^u \\ = r\sqrt{\frac{1}{3}} \left(a_1 + a_2 X + \frac{3}{2}(a_9 - a_7)X + \frac{3}{2}(a_{10} + a_8 R_4) \right) \quad (30)$$

$$a_{3/2}^c e^{i\delta_{3/2}} = \sqrt{\frac{1}{3}}A_{\pi^+ K^-}^c - \sqrt{\frac{2}{3}}A_{\pi^0 \bar{K}^0}^c \\ = r\frac{\sqrt{3}}{2}(a_{10} + a_8 R_4 + (a_9 - a_7)X) \quad (31)$$

and

$$\sqrt{\frac{2}{3}}b_{1/2}^c - \frac{1}{\sqrt{3}}a_{3/2}^c = A_{\pi^- \bar{K}^0}^c \\ = r \left(a_4 + a_6 R_5 - \frac{1}{2}(a_{10} + a_8 R_5) \right) \quad (32)$$

where $r = i\frac{G_F}{\sqrt{2}}f_K F_0^{B\pi}(m_K^2)(m_B^2 - m_\pi^2)$ and $X = (f_\pi/f_K)(F_0^{BK}/F_0^{B\pi})(m_B^2 - m_K^2)/(m_B^2 - m_\pi^2)$. In our numerical estimates, we will take $f_\pi = 133$ MeV, $f_K = 158$ MeV, $F_0^{B\pi} = 0.36$ and $F_0^{BK} = 0.41$. There remains a large uncertainty in strange quark mass m_s . For $m_s = (100 \sim 200)$ MeV, we find that the numerical values of those amplitudes are given by

$$\begin{aligned} a_{1/2}^u &\simeq 818 \sim 846, \quad a_{3/2}^u \simeq 709, \\ a_{1/2}^c &\simeq -(103 \sim 131), \quad a_{3/2}^c \simeq -7, \\ b_{1/2}^c &\simeq -(72 \sim 100). \end{aligned} \quad (33)$$

The results from the χ^2 -fitting are shown in Fig.1-3, where the six amplitudes as well as their errors at 1σ level are obtained as functions of the weak phase γ with δ' fixed at $0, \pi/6, \pi/3$ respectively. The relative magnitudes of the amplitudes are consistent with the previous discussions. In our fit, the minimum value of χ^2 is found to be extremely low (typically $\chi_{min}^2 \sim 0.5 \times 10^{-12}$). This means that the χ^2 -fits are highly consistent and the 6 amplitudes are actually extracted as the solutions of Eqs.2-8. It can be seen from the figures that the γ dependence of $a_{1/2}^u$ is quite strong and the one of $a_{1/2}^c$ and $a_{3/2}^c$ is relatively weak. On the other hand, it may be used to extract the angle γ once one of those amplitudes can be determined or calculated in other independent ways. It is of interest to see that the γ dependence of the amplitude $b_{1/2}^c$ and the strong phase δ is weak, which shows that these two quantities are approximately fixed. The possibility of large δ was also suggested in Ref. [4] to explain the large branching ratio of the $B \rightarrow \pi^0 K^0$ decay. Recently, perturbative QCD calculations have also shown a large strong phase [17].

In Fig.1 where the phase difference δ' is set to be zero as usual, the χ^2 -fitting shows that the values of $a_{1/2}^u$ and $a_{3/2}^u$ may be comparable with the ones from the theoretical estimations. only when the weak phase γ is large. Especially for $\gamma > 2\pi/3$, the fitting values could coincide with the ones from factorization except for $a_{3/2}^c$. For $\gamma < 2\pi/3$, the two amplitudes are smaller than the ones from naive factorization calculations. It appears that the factorization approach may become suitable for large weak phase γ . This phenomenon was observed by most of the analyses in the literature which neglects the isospin amplitude $a_{3/2}^c$. As a consequence, the resulting large value of γ seems to be conflict with the one obtained from other constraints in the standard model. Before drawing the final conclusion, one also notice that in the factorization approach, one yields a zero strong phase $\delta = 0^\circ$ which actually contradicts with the general fitting value $\delta \simeq \pm 95^\circ$. Therefore, the results of estimates based on the naive factorization approach should be unreliable, and the isospin amplitudes must receive additional large contributions. A large value for the relative strong phase $\delta \pm 95^\circ$ implies that the final state interactions or inelastic rescattering effects must be significant.

We would like to stress that the most outstanding feature of the χ^2 analysis with $\delta' = 0$ is that the isospin amplitude $a_{3/2}^c$ is likely to be relatively larger than the one estimated from the naive factorization calculations. The fitting central value of $a_{3/2}^c$ may be larger by a factor of $7 \sim 9$. To explicitly see how the decay rates depend on the isospin amplitude $a_{3/2}^c$, we plot in Fig.4 the 6 branching ratios of $B \rightarrow \pi\pi, \pi K$ decays as functions of $a_{3/2}^c$. It can be clearly seen that if $\delta' = 0$, a small value of $a_{3/2}^c \sim -7$ is not able to reproduce all the CLEO data within the 1σ level, especially the data for the channels of $\pi^0 \bar{K}^0$ and $\pi^0 K^-$. To consistently describe the whole data, we need a relative large value $a_{3/2}^c \sim -75$ for fitting the central value of the data which is about 10% of the largest one a_0^u .

Within the standard model, it seems difficult to enhance the isospin amplitude $a_{3/2}^c$ by an order of magnitude even the inelastic FSI is involved, this is because the main inelastic channels such as $B \rightarrow DD(DD_s, \eta_c K) \rightarrow \pi\pi(K)$, only contribute to isospin- $\frac{1}{2}$ part of the decay amplitude. In the SM, it is known that the ratio $a_{3/2}^c/a_{3/2}^u$ can be determined without the hadronic uncertainties in the flavor $SU(3)$ limit, this is because the ratio only depends on the short distance Wilson coefficients [19]. Thus a large value of $a_{3/2}^c/a_{3/2}^u$ may indicate the existence of new physics. While all models beyond the standard model must effectively provide large contributions to the electroweak penguins in order to enhance the isospin amplitude $a_{3/2}^c$, such models are: SUSY with R parity violation, Z' -model and Z -mediated FCNC models, et. al.

Let us now consider the case that δ' is non-zero, the situation then becomes quite different. In Fig.2 and Fig.3, it is seen that $a_{3/2}^c$ decreases as the value of δ' increases. When δ' reaches $\pi/3$, $a_{3/2}^c$ will be consistent with the value yielded from factorization approach. It is also noticed from the figures that a large δ' leads to large values of $a_{1/2}^u$ and δ . The enhancement of $a_{1/2}^u$ may be easily understood than the enhancement of $a_{3/2}^c$ due to final state interactions. Nevertheless, as will be discussed below, the values of δ' cannot be too large due to the constraint of the upper bound of the branching ratio of $B \rightarrow \pi^0\pi^0$.

When all the isospin amplitudes and strong phases are determined, one is able to predict the direct CP asymmetries for all the relevant decay channels. The direct CP asymmetry in $B \rightarrow \pi\pi, \pi K$ decays is defined in the standard way

$$A_{CP} = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)} \equiv a_{\epsilon''}^f \quad (34)$$

where f denotes the final state mesons. In Fig.5 we plot several $|A_{CP}|$'s as functions of the weak phase γ with different value of δ' . When γ is near 90° and $\delta' = 0$ one has $|A_{CP}(\pi^+K^-)| \sim 0.04$ which is in a good agreement with the most recent CLEO data $A_{CP} = -0.04 \pm 0.16$ [2]. At this point, we have a reliable prediction for the direct CP violations in the following decay modes. The A_{CP} 's with $45^\circ < \gamma < 95^\circ$ read

$$\begin{aligned} |A_{CP}(\pi^+K^-)| &\simeq (2.5 \sim 4)\% , & |A_{CP}(\pi^0\bar{K}^0)| &\simeq (2.5 \sim 5)\% , \\ |A_{CP}(\pi^0K^-)| &\simeq (7.5 \sim 10)\% , & |A_{CP}(\pi^-\bar{K}^0)| &\simeq (5 \sim 6.5)\% , \\ |A_{CP}(\pi^+\pi^-)| &\simeq (7.5 \sim 12.5)\% , & |A_{CP}(\pi^0\pi^0)| &\simeq (7.5 \sim 12)\% . \end{aligned} \quad (35)$$

for $\delta' = 0$, and

$$\begin{aligned} |A_{CP}(\pi^+K^-)| &\simeq (5 \sim 8)\% , & |A_{CP}(\pi^0\bar{K}^0)| &\simeq (8 \sim 10)\% , \\ |A_{CP}(\pi^0K^-)| &\simeq (12 \sim 18)\% , & |A_{CP}(\pi^-\bar{K}^0)| &\simeq (8 \sim 12)\% , \\ |A_{CP}(\pi^+\pi^-)| &\simeq (16 \sim 24)\% , & |A_{CP}(\pi^0\pi^0)| &\simeq (10 \sim 14)\% . \end{aligned} \quad (36)$$

for $\delta' = 30^\circ$. In spite of the large δ , the smallness of CP asymmetries in $B \rightarrow \pi^-\pi^0$ decay is due to the absence of the interference between tree and penguin diagrams.

There remains an unobserved decay mode in $B \rightarrow \pi\pi, \pi K$ decays, which is the decay mode $B \rightarrow \pi^0\pi^0$ (the CLEO collaboration has already reported the indication of $Br(B \rightarrow \pi^0\pi^-) \simeq 5.6$). As all the relevant isospin amplitudes have been determined as functions of γ , it allows us to predict $B \rightarrow \pi^0\pi^0$ as a function of γ . It is interesting to note that our

χ^2 -analysis shows that the resulting branching ratio $Br(B \rightarrow \pi^0\pi^0)$ is almost independent of the weak phase γ . Its value at $\delta' = 0$ is close to the one of $B \rightarrow \pi^+\pi^-$ decay

$$Br(B \rightarrow \pi^0\pi^0) \sim 4.6 \times 10^{-6}. \quad (37)$$

with δ' increasing, the branching ratio becomes larger and can reach $\sim 7(10) \times 10^{-6}$ when $\delta' = 30^\circ(60^\circ)$. Such a large branching ratio is about an order of magnitude larger than the prediction based on factorization calculations. Most recently, the CLEO collaboration reported an upper bound of $Br(B \rightarrow \pi^0\pi^0) < 5.7 \times 10^{(-6)}$ [21], this will impose a strong constraint on the value of δ' (see Fig. 6). It is seen that to be consistent with the data at the 1σ level, the upper bound of the branching ratio $Br(B \rightarrow \pi^0\pi^0)$ limits δ' to be less than $\sim 50^\circ$.

IV. CONCLUSIONS

In summary, we have made a general less model-dependent investigation on the charmless B-meson decays by using the χ^2 analysis based on the most recent CLEO data. We have used the most general isospin decomposition with two independent strong phases. All the isospin amplitudes in rare hadronic B decays $B \rightarrow \pi\pi, \pi K$ can be determined as functions of the weak phase γ and one strong phase δ' . The effects of two equal and unequal strong phases are studied in detail. It is found that the isospin amplitude $b_{1/2}^c$ and the strong phase δ only slightly depend on the phase γ . An important observation under the equal strong phase assumption is the relative large isospin amplitude $a_{3/2}^c$, ($a_{3/2}^c \simeq -70$) where the central value is about 5 times greater than the one obtained from the factorization calculations. When the two strong phases are not equal, the allowed values of $a_{3/2}^c$ decrease as their difference, i.e., δ' , increases. For the most general case with two rather than one large FSI strong phases, the magnitude of all the isospin amplitudes may be around the one estimated from the factorization approach. Nevertheless, one needs to find out the mechanism of producing large strong phases. This could directly be tested by measuring the branching ratio $Br(B \rightarrow \pi^0\pi^0)$.

The direct CP asymmetries A_{CP}^f for all the relevant decay channels have also been given as functions of γ . The resulting numerical value for $A_{CP}^{\pi^+K^-}$ is consistent with the most recent data. When taking γ to be in a reasonable range $\gamma = 45^\circ \sim 95^\circ$, we are led to the results given in eqs.(35) and (36), which can be directly tested by experiments in the near future. A resulting large branching ratio $Br(B \rightarrow \pi^0\pi^0)$ which is comparable with the one $Br(B \rightarrow \pi^+\pi^-)$ will also provide an important and consistent test.

From the most general analysis presented in this paper, the data appears to strongly suggest that final state interactions and inelastic rescattering effects must be significant and play an important rule in the charmless $B \rightarrow \pi\pi, \pi K$ decays. Otherwise, our general analyses may be interpreted as hinting at the existence of new physics. For a more definite conclusion, one needs more precise data. The two B-factories BaBar and BELLE are expected to provide us with more information from the charmless decays.

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REFERENCES

- [1] CLEO Collab. hep-ex/0001010.
- [2] CLEO Collab. hep-ex/0001009.
- [3] G. Kramer, W.F.Palmer *Phys. Rev.* **D52** (1995) 6411.
N. G. Deshpande and X. G. He, *Phys. Rev. Lett.* **74** (1995) 26.
- [4] W. S. Hou and K. C. Yang hep-ph/9911528, to appear in *Phys.Rev.Lett.*
- [5] R. Fleischer, *Eur. Phys. J.***C6**(1999)451.
- [6] M. Gronau and J. L. Rosner *Phys. Rev. Lett.* **73** (1994) 21
M. Gronau and J. L. Rosner *Phys. Rev. Lett.* **76** (1996) 1200 .
- [7] M. Neubert and J. L. Rosner *Phys. Rev. Lett.* **81** (1998) 5076.
- [8] N. G. Deshpande, X. G. He, W. S. Hou and S. Pavaska *Phys. Rev. Lett.* **82** (1999) 2240;
X. G. He, W. S. Hou and K. C. Yang *Phys. Rev. Lett.* **83** (1999) 1100;
W. S. Hou and K. C. Yang hep-ph/9908202 .
- [9] T. N. Pham, hep-ph/0009142,
C. Isola, T. N. Pham, hep-ph/9911534.
- [10] Y. L. Wu and Y. F. Zhou, *Phys. Rev.* **D62** 036007 (2000).
- [11] X. G. He, C. L. Hsueh and J. Q. Shi *Phys. Rev. Lett.* **2000** (84) 18;
Y.L. Wu and Y.F. Zhou, *Phys. Rev.* **D61** 096001 (2000).
- [12] D. Zppenfeld, *Zeit. für Physik* **C8** (1981) 77;
M. Savage and M. Wise, *Phys. Rev.* **D39** (1989) 3346;
M. Savage and M. Wise, *Phys. Rev.* **D40** (1989) 3127(E);
M. Gronau, O. F. Hernandez, D. London and J. L. Rosner , *Phys. Rev.* **D52** (1995) 6374;
M. Gronau, O. F. Hernandez, D. London and J. L. Rosner , *Phys. Rev.* **D52** (1995) 6356;
X.G. He, *Eur. Phys. J.* **C9** (1999) 443;
M. Gronau and J. L. Rosner *Phys. Rev.* **D61** (2000) 073008.
- [13] R. Fleischer and T. Mannel, *Phys. Rev.* **D57** (1998) 2752.
- [14] A.J. Buras and R. Fleischer hep-ph/0003323.
- [15] M. Gronau and J. L. Rosner *Phys. Rev.* **D61** (2000) 073008.
- [16] F.James and M.Roos, *Compu.Phys.Comm.***10**,343(1975)
CERNLIB-writeup, <http://wwwinfo.cern.ch/asdoc/shortwrupsdir/d506/top.html>
- [17] Y.Y. Keum, H.-N. Li and A.I. Sanda, Talk given at 28th Intern. Workshop on Gross Properties of Nuclei and Nuclear Excitation: Hadrons in Dense Matter (HIRSCHEGG 2000), Hirschegg, Austria, 16-22 Jan 2000, hep-ph/0004173.
- [18] A. Ali, G. Kramer and C.D. Lü, *Phys. Rev.* **D58** (1998) 094009.
- [19] M.Neubert, J.Rosner, *Phys. Lett.* **B441** (1998) 094009.
- [20] D.Hitlin, et al, BaBar collaboration, plenary talk in ICHeP 2000(Osaka, Japen,July 21,2000), SLAC-PUB-8540.
H.Aihara, Belle collaboration, plenary talk in IC HeP 2000(Osaka, Japen,July 21,2000),
- [21] CLEO collaboration, plenary talk in ICHEP 2000(Osaka, Japen,July 21,2000).

FIGURES

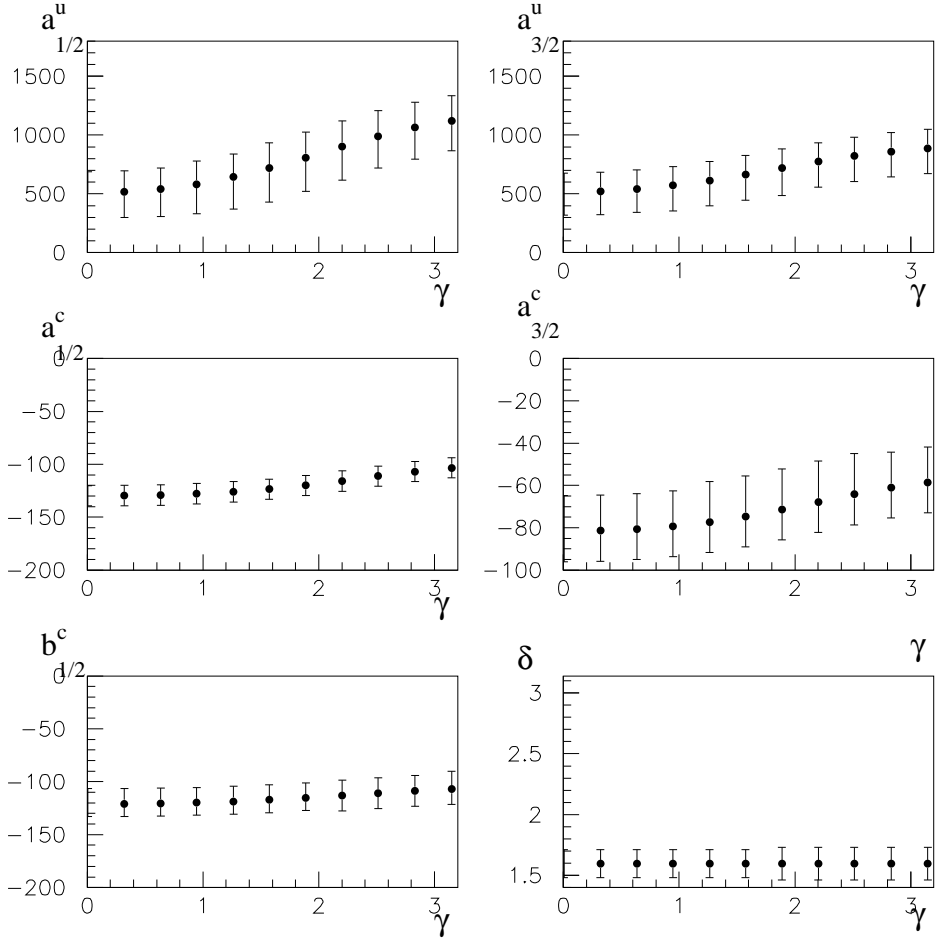


FIG. 1. The isospin amplitude fitted as a function of weak phase γ from a χ^2 - analysis of the recent CLEO data. The vertical bars indicate the errors at 1σ level. the strong phase δ' is set to be zero.

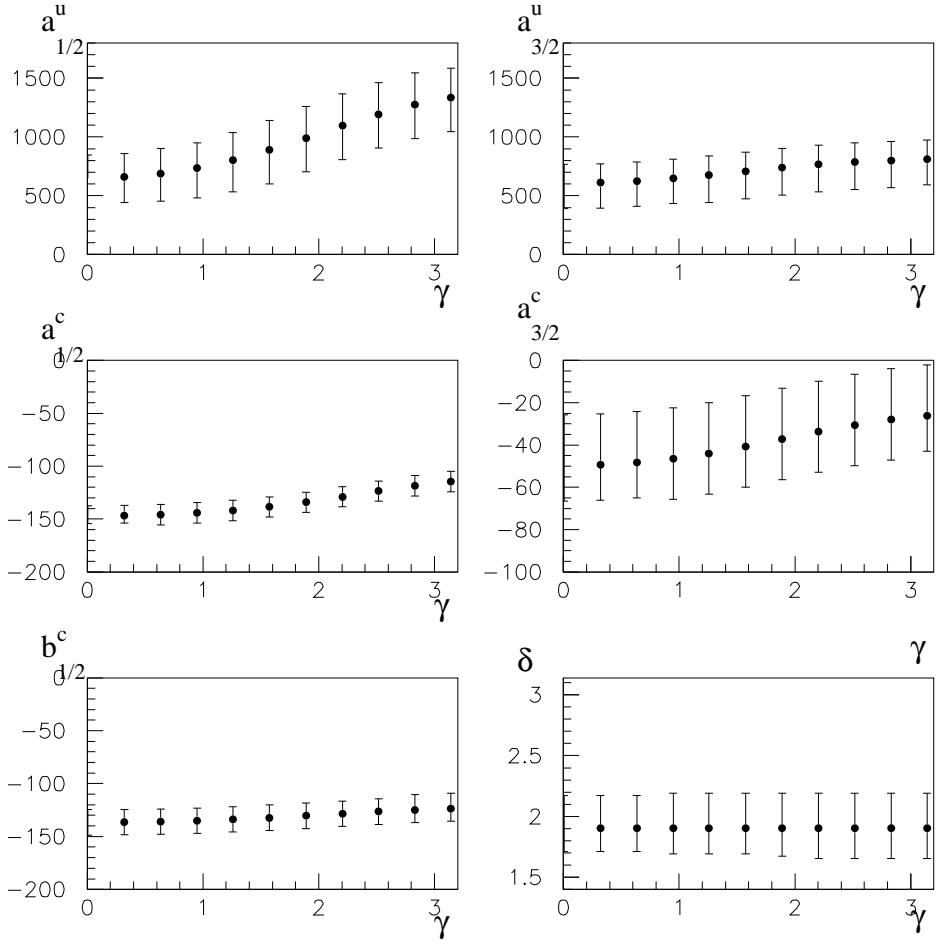


FIG. 2. The same as Fig.1 but with $\delta' = \pi/6$

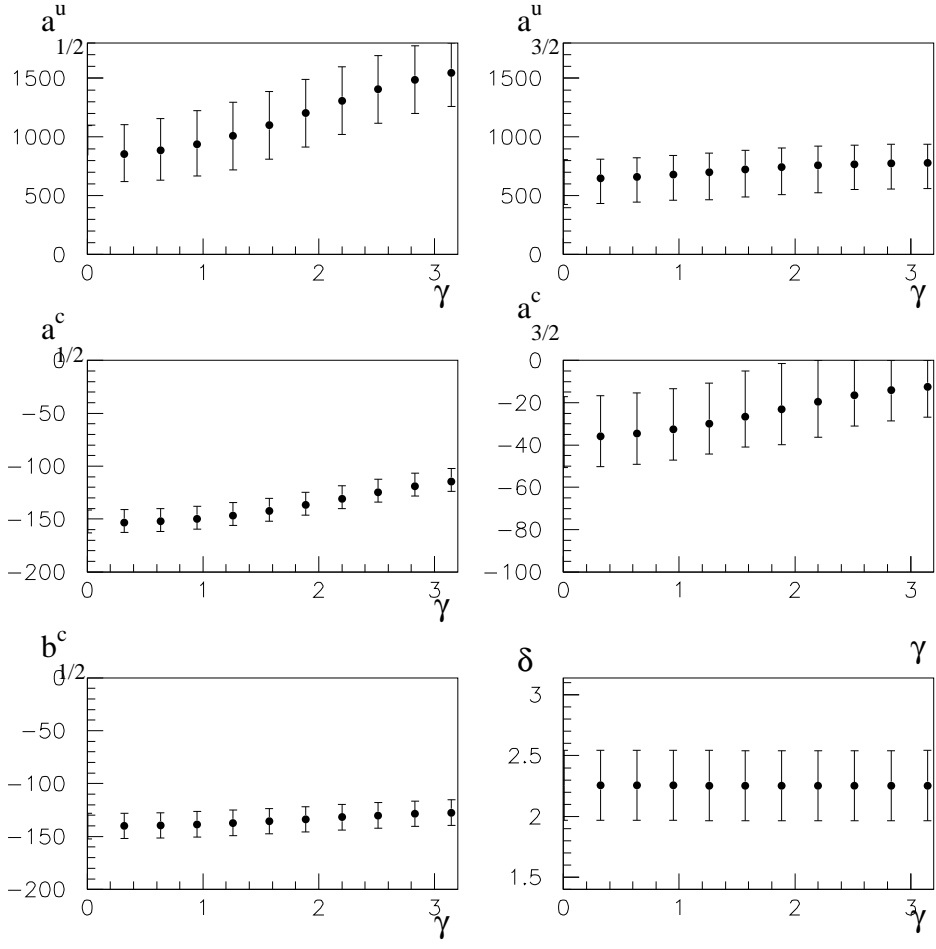


FIG. 3. The same as Fig.1 but with $\delta' = \pi/3$

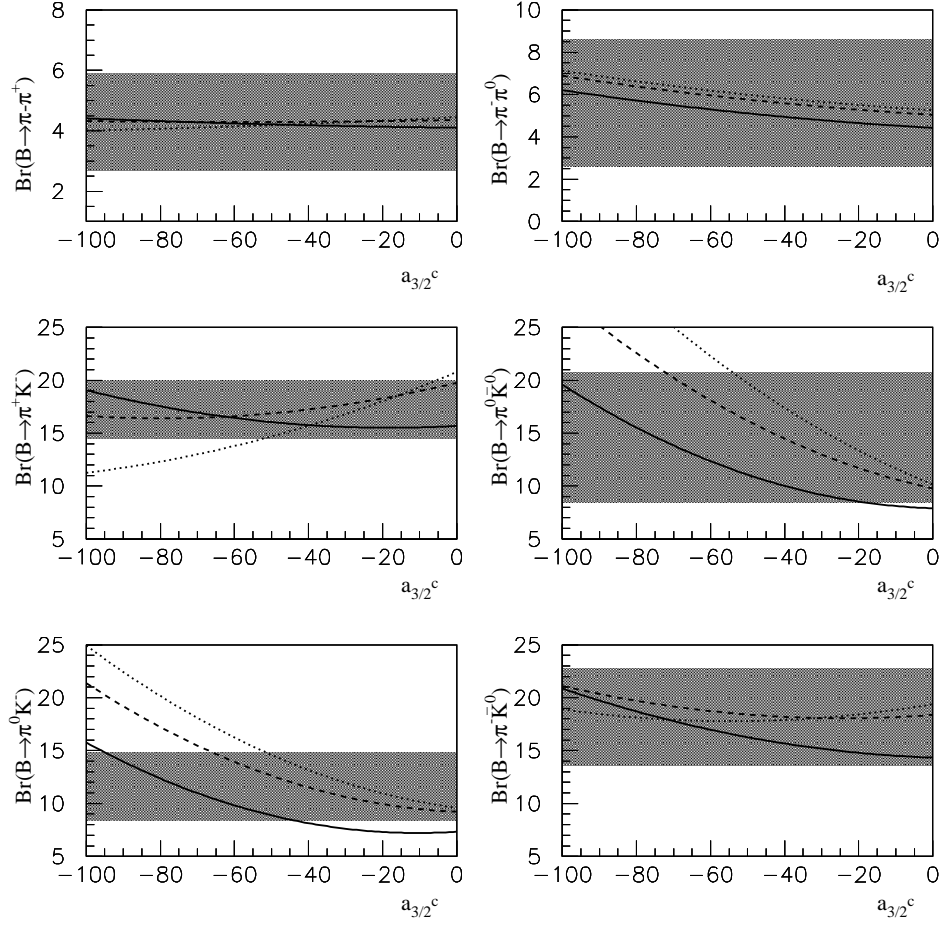


FIG. 4. The $a_{3/2}^c$ dependence of the 6 branching ratios(in units of 10^{-6}), for $\gamma = 70^\circ$. The other parameters are at their central values. The three curves in each plot correspond to $\delta' = 0$ (solid), $\pi/6$ (dashed), $\pi/3$ (dot-dashed) The hashed bands indicate the errors(at 1σ) of the data.

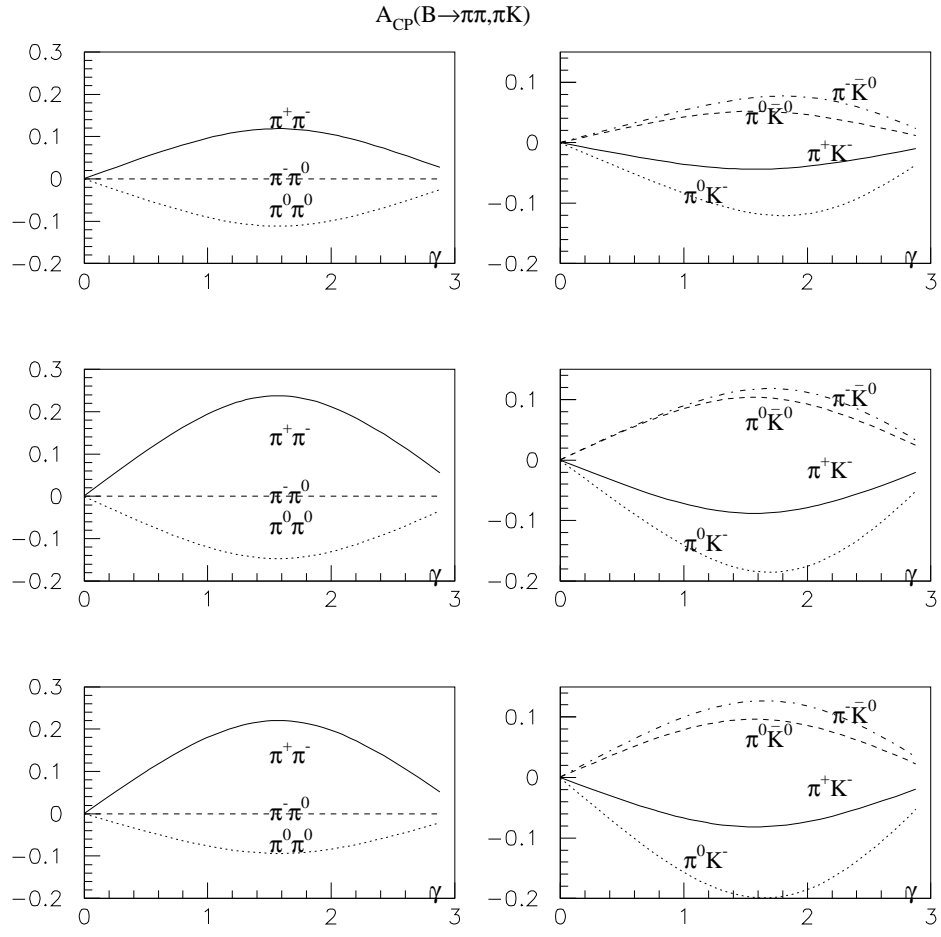


FIG. 5. The values of CP asymmetries A_{CP} vs weak phase γ with different $\delta' = 0, \pi/6, \pi/3$ (from up to down) The curves corresponding $|A_{CP}|$ s for π^0K^- , $\pi^+\pi^-$, $\pi^0\bar{K}^0$, π^+K^- , $\pi^-\bar{K}^0$ and $\pi^-\pi^0$ are indicated in the plot.

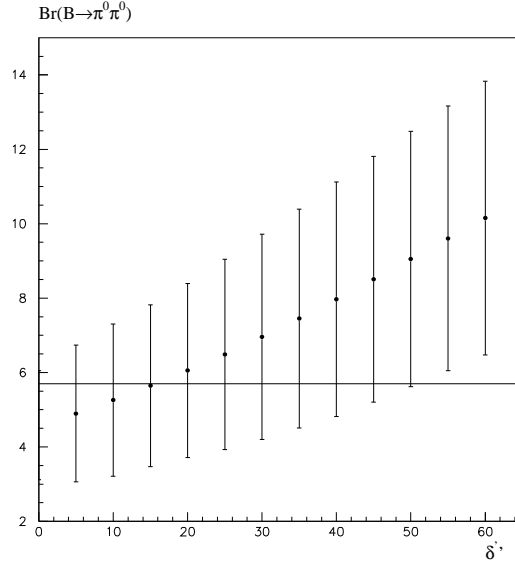


FIG. 6. the branching ratio of $B \rightarrow \pi^0 \pi^0$ (in units of 10^{-6} predicted as function of δ' (in degree). the solid line indicate the upper bound observed reported by CLEO collaboration.